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Constructing Formal Semantics from an Ontological Perspective. The Case of Second-Order Logics

Abstract :

In a first part, I defend that formal semantics can be used as a guide to ontological commitment. Thus, if one endorses an ontological view O and wants to interpret a formal language L, a thorough understanding of the relation between semantics and ontology will help us to construct a semantics for L in such a way that its ontological commitment will be in perfect accordance with O. Basically, that is what I call *constructing formal semantics from an ontological perspective*. In the rest of the paper, I develop rigorously and put into practice such a method, especially concerning the interpretation of second-order quantification.

I will define the notion of *ontological framework*: it is a set-theoretical structure from which one can construct semantics whose ontological commitments correspond exactly to a given ontological view. I will define five ontological frameworks corresponding respectively to: (i) predicate nominalism, (ii) resemblance nominalism, (iii) armstrongian realism, (iv) platonic realism, and (v) tropism.

In those different frameworks, I will construct different semantics for first-order and second-order languages. Notably I will present different kinds of nominalist semantics for second-order languages, thus showing how we can quantify over properties and relations while being ontologically committed only to individuals. More generally I will show in what extent those semantics differ from each other; it will make clear how the disagreements between the ontological views extend from ontology to logic, and how metaphysical questions can be correctly treated, in those semantics, as simple questions of logic.

Keyword: ontology, semantics, ontological commitment, second-order logic, nominalism, realism, tropes.

1. Formal semantics as a guide to ontological commitment

1.1. Formal semantics as describing a truthmaking structure

There are basically two steps in constructing a formal semantics for a language:

i) First we have to define what is a *model* for this language. (It is usually defined by set-theoretical conditions.)

ii) Then we give rules according to which, for every model and every formula of the language, we can determine whether or not this formula in this model has a certain "value". (In the simplest case, there is only one value: true.)

Is formal semantics a legitimate source of information about ontology? One could argue that it is nothing more than an abstract device whose purpose is to prove certain features of a deductive system, especially its consistency. This passage from Zalta [1] illustrates perfectly this attitude towards semantics:

> "It is important to remember that the formal semantics simply provides a set-theoretical framework in which models of the metaphysical theory may be constructed. The models serve the heuristic purpose of helping us to visualize or picture the theory in a rigorous way. It is extremely important not to confuse the models of the theory with the world itself. Many theorist today tend to give models of a theory an exalted status that they do not have. (…) As far as the present work is concerned, all that the models of a theory do is show that the theory is consistent, that the logic is complete, that the axioms are categorical, and so forth." (Zalta [1], p.34-35)

According to such a view, a model does not represent in any way the structure of a *world*, and therefore the defined notion of *truth in a model* has in fact nothing to do with truth; semantics is nothing but a logical tool without any ontological significance. This a convenient view for a neo-meinongian theorist such as Zalta: even if the models of his theory contains a domains of objects, counting among them impossible, contradictory and incomplete objects, since those models are not supposed to be an image of the world, one could endorse the theory while refusing (or at least not endorsing openly) the idea that this domain of objects is *really* in our world.

There is nothing wrong in considering semantics in that reductive way. But now let me explain why I think we should give to formal semantics a more interesting purpose.

I am a realist in the following sense: I assume that the world (the way things are in the world) is what makes true the true propositions of our language. If one accepts this realist premise, the idea that formal semantics may give a picture of this world/language relation is very appealing: a model seems to represent the structure of a world, and the semantic rules give an account of how any given world makes true or false every formula of the language.

Thus, from a realist point of view, a natural purpose of formal semantics would be to describe the world/language relation.

Note that talking about worlds for models may be misleading for the following reason: in semantics for modal languages (often called *possible worlds* semantics), certain elements of the models are supposed to stand for worlds or possible worlds, therefore it would be inappropriate to think of a model as a world itself. One might prefer to say that such a model represents the structure of a *universe* of possible worlds; thus the semantic rules give an account of how this modal universe makes true or false every formula of the modal language. There is a more neutral option anyway, if one does not want to talk about worlds nor universes: one might say that a model represents the structure of an *ontological situation* in general, i.e. any possible answer to the ontological question: what is there?

In conclusion, if we take semantics seriously, i.e. if we take it as an attempt to describe the world/language relation, to define what is a *model* is basically to define what is the structure of any ontological situation; and the *semantic rules* are rules governing how any ontological situation makes true or false any formula of the language. In other terms, the semantic rules describe the *truthmaking* of the formulas.

1.2. Truthmaking as a guide to ontological commitment

I will defend that the ontological commitments of a sentence or a theory are to be read off what *makes true* this sentence or this theory (granted that they are true). Such a view as been endorsed by various authors, see for example Simons [2], Heil [3], Armstrong [4], Cameron [5, 6]. Though it offends the quinean orthodoxy about ontological commitment (but we may note that many philosophers have already argued against it, see for example the excellent criticism in Prior [7] ch.3 about second-order quantification), I think that the position I defend is in fact very natural and intuitive as soon as one adopts a friendly attitude towards the idea of truthmaking. This passage of Simons [1997] described very accurately how the two notions of truthmaking and ontological commitment seem to be related:

> "Ontological commitment is a sort of converse to an idea which is of more recent prominence: truth-making. Whereas when we ask what things are such that their existence is *necessary* for a sentence to be true, we are asking after its ontological commitments; when we ask what things are such that their existence is *sufficient* for the sentence to be true, we are considering the sentence's *truth-makers*. (...) We could therefore characterize the ontological commitments of a sentence or sentences as given by the least that would be required to make it true." (Simons [2] p.265.)

The quantificational approach and the truthmaking approach would not be in opposition if the truthmaker of an existentially quantified sentence like "there is X" were always that very X. But a truthmaker theorist can precisely reject this idea. As Cameron puts it:

> "I think one of the benefits of truthmaker theory is to allow that $<x$ exists$>$ might made true by something other than x, and hence that 'a exists' might be true according to some theory without being an ontological commitment of that theory." (Cameron [5], p.401)

Let us take an example. According to the quinean criterion, if the sentence "There is a table" is literally true, we are ontologically committed to the existence of a distinct entity corresponding to the table. Thus, if we think that a table is a bunch of atoms arranged in a certain way, we must not take "There is a table" as literally true but as an improper way to say "There are such and such atoms arranged in such and such way". On the other hand, according to the truthmaking criterion, "There is a table" ontologically commits us only to *whatever makes true this sentence*. (More precisely, we must consider a *minimal* truthmaker.) If we assume that a (minimal) truthmaker for "There is a table" is only the fact that a bunch of atoms are arranged in such and such way, then this sentence only commits us to the existence of these atoms arranged in that way. Thus, "There is a table" can be literally true without committing to the existence of a new entity.

Another insightful example is given by Simons:

> "This way of looking at ontological commitment highlights something which might otherwise remain clouded, and which one might call the inscrutability of ontological commitment. Consider first a simple medical sentence such as *Sam has hepatitis*. This is made true by hepatitis viruses in sufficient numbers in Sam's body, causing inflammation of his liver. But we cannot tell this by simply analyzing the sentence linguistically: it is a matter of medical knowledge, not conceptual analysis. Not even the type of virus is fixed by the statement: when in a paper on truth-making, Mulligan *et al.* put forward this example, two types of hepatitis virus were known, since then another has come to light, which only serves to underline the point that it is in general an *a posteriori* matter what makes a given sentence true. But if that is so then the ontological commitments or truth-making minima of a sentence are not to be read off its logico-grammatical form either: when we affirm that Sam has hepatitis then all we are committed to is *something or other, whatever it is* that causes Sam's liver to be inflamed. In general the sentences whose commitments are most readily ascertainable, at least in part, are existential assertions, whether particular or general, but they obviously form only a small proportion of all sentences." (Simons [2] p.265.)

Of course, this approach of ontological commitment is not exempt of difficulties, but I think it is the best way to ask the question: most clearly if we are realist about truth, there is a strong connection between what *would make true* a theory and *what there would be* according to this theory, in other terms between the truthmakers of the theory and its ontological commitments.

Now, if we take formal semantics seriously, that is to say if we think that the semantics of a formal language provides an account of the truthmaking of every formula of this language in any ontological situation, and if we adopt the truthmaker approach for ontological commitment, then we have excellent reason to take formal semantics as a guide to ontological commitment.

Does it mean that we must follow the semantics wherever it leads us from an ontological point of view? No. The idea is precisely to proceed in the other way. As soon as we will understand correctly the relation between formal semantics and ontology, it will become possible to construct a semantics in such a way that *in fine* its ontological commitments will be in accordance with a given ontological view. That is what I call *constructing semantics from an ontological perspective*.

I will now put into practice this method.

2. Five ontological frameworks

In semantics theory we usually define a model in a set-theoretical framework. A model for a language $\mathcal{L}_{\mathcal{V}}$ constructed from a set of non-logical constant $\mathcal{V}$ (its vocabulary) will be most of the time a structure of this form $<X, Y, \ldots, \delta>$ where *X*, *Y*, … are any sort of sets or functions, and δ is a *denotation function*, i.e. a function mapping elements of the vocabulary $\mathcal{V}$ to something in the model. Except this function δ which assures the interpretation of the language, every other element of a model can be seen as purely *ontological elements*. Hence, what I call the *ontological framework* from which a semantics is constructed, it is the structure $<X, Y, \ldots,>$, i.e. the structure of a model without the denotation function.

The ontological framework is supposed to represent the structure of *what there is*, the structure of any ontological situation. Thus, choosing a framework or another should only reflect ontological insights, independently from the languages we want to interpret in this framework.

Things will get clearer with few examples. Let us consider five classical ontological views about the status of individuals and universals:

Strong nominalism. The world is made of individuals and nothing else, and it is not structured in any way.

Weak nominalism. The world is made of individuals and nothing else; however, the set of individuals is structured. (For example it can be structured by a resemblance relation.)

Strong nominalism agrees with weak nominalism about the fundamental bricks of reality: only individuals. But according to strong nominalism, this set of bricks has no structure; a predicate like *Red* correspond to nothing in reality; an individual is red only because this predicate applies to it. This view is also called *predicate nominalism*. On the contrary, according to weak nominalism, the use of a predicate may be grounded in an ontological structure. The most popular candidate is resemblance: there are only individuals, but they are structured by a resemblance relation. The predicate *Red* applies to an individual because this individual belongs to a set of resembling things.

Weak realism. What is fundamental is the state of affairs *a universal n-adic being instantiated by n individuals*. Therefore there are individuals and universals, but they are always connected in a state of affairs.

Strong realism. There are universals, and there being a given universal is independent from there being any individual instantiating them. On the contrary, individuals only exists as they instantiate universals.

Weak realism corresponds to a certain kind of aristotelianism, a view being notably held by David Armstrong. Strong realism corresponds to a full-blooded platonism about universals.

Tropism. The only fundamental entities are *tropes* or *abstract particulars*. Individuals are constituted of compresent tropes and universal are constituted of resembling tropes. About tropism, see 2.5.

None of these five claims tells us specifically what there is; they only tell us what *kind* of things there are. In other terms, it is an answer to the question "what is the structure of an ontological situation?", not an answer to the question "what is the actual ontological situation?".

It is easy to see that an ontological framework corresponds to an answer to the first question: "what is the structure of an ontological situation?" Indeed, as I defined it, an ontological framework determine a certain kind of models, and those models are supposed to represent ontological situations. Thus, those five ontological views can be represented as five different ontological frameworks.

In the rest of this section I will show how we can define those frameworks and I will give simultaneously for each of them a sketch of a semantics for first-order languages. Then in section 3, I will present in full details different semantics for second-order languages constructed from each of those different frameworks.

2.1. Strong nominalist framework

The strong nominalist framework is the simplest. There are only individuals, and they are not structured in any way, therefore this is simply a structure $<\mathcal{I}>$ where $\mathcal{I}$ is a set of urelements (standing intuitively for individuals).

Thus, a nominalist semantics is a semantics in which models have the following structure: $<\mathcal{I}, \delta>$. In standard semantics for first-order logic, models have precisely this structure; a constant object a denotes a member of $\mathcal{I}$, a monadic predicate P denotes a set of members of $\mathcal{I}$, and the formula Pa is true iff $\delta(a) \in \delta(P)$. Thus we can properly say that standard semantics for first-order logic is a strong nominalist semantics.

Note that the framework represents the *structure* of what there is; not what there is actually. I am not saying that according to a strong nominalist, the world *is* a set (which would be clearly false); I am just saying that the structure of the world is adequately represented as a set.

I will call the urelements of a framework the *entities* of this framework. In a strong realist frameworks, individuals are the only entities. (In a certain sense, entities can be considered as the *building bricks of the world*, but that does not mean that the set of entities is fundamental; I will soon say more about fundamentality and we will see that there is a difference between being an entity and being fundamental.)

2.2. Weak nominalist framework

In a weak nominalist framework, there are still only individuals, but the set of individuals is structured. Let us assume that it is structured by *resemblance*. (It is the most standard account. We could also consider the idea of *natural class*. I do not think the ontological framework would be very different.) Therefore, we need something more than just a set $\boldsymbol{\mathcal{I}}$ of individuals.

Let us first define a weak nominalist framework as a structure $\langle \boldsymbol{\mathcal{I}}, \boldsymbol{\mathcal{R}} \rangle$ where $\boldsymbol{\mathcal{I}}$ is a set of urelements, let us call them the individuals, and $\boldsymbol{\mathcal{R}}$ is a set $\{\boldsymbol{\mathcal{R}}_1, \ldots, \boldsymbol{\mathcal{R}}_n, \ldots\}$ where each $\boldsymbol{\mathcal{R}}_n$ is a set of sets of *n*-tuples of individuals, and such that each *n*-tuple of individual appears in at least one member of $\boldsymbol{\mathcal{R}}_n$. Intuitively, $\boldsymbol{\mathcal{R}}_n$ is a set whose members are a sets of resemblance, i.e. sets of *n*-tuples of individuals resembling each other in some aspect. (Note that the empty set may be a member of $\boldsymbol{\mathcal{R}}_n$.) If we consider the set $\boldsymbol{\mathcal{R}}_1$, this set may contain for example a set of things containing apples, tomatoes, etc., things resembling each other in their redness. The condition that each *n*-tuple of individuals appears in at least one member of $\boldsymbol{\mathcal{R}}_n$ expresses the reflexivity of resemblance: each *n*-tuple of individuals resembles at least itself. (The symmetry is assured by the fact that we are considering sets of resembling things. But we do not have transitivity since those sets may partially overlap.)

I will call *basic sets* of a framework a set whose construction cannot be entirely determined by another element of the framework. This notion will be useful in order to apprehend what is *fundamental* in a given framework. In the weak nominalist framework, $\boldsymbol{\mathcal{I}}$ is not a basic set: it can be constructed from any of the $\boldsymbol{\mathcal{R}}_n$'s. Therefore we could rather take $\langle \boldsymbol{\mathcal{R}} \rangle$ as the weak nominalist framework, where $\boldsymbol{\mathcal{R}}$ is a set $\{\boldsymbol{\mathcal{R}}_1, \ldots, \boldsymbol{\mathcal{R}}_n, \ldots\}$ such that: $\boldsymbol{\mathcal{R}}_1$ is a set of sets of urelements; we define the set $\boldsymbol{\mathcal{I}}$ as the union of these sets, and let us call those urelement *individuals*; and the other $\boldsymbol{\mathcal{R}}_n$'s (for $n \geq 2$) are defined as previously.

What is basic in the weak nominalist framework, thus, is not a set of individuals but a structure of resemblances on these individuals. Those individuals however are still the entities of the framework (they are the urelements). Therefore we can say that strong nominalism and weak realism agree on the entities, but not on the fundamental structure of reality: for the former, it is nothing but an unstructured set; for the latter, reality is structured by resemblance.

A weak nominalist semantics for first-order languages is not exactly standard but it is very close. A model would have to be a structure $<\boldsymbol{\mathcal{R}}, \delta>$ (and not $<\boldsymbol{\mathcal{I}}, \delta>$, that is the only difference); as in standard semantics, a constant object denotes a member of $\boldsymbol{\mathcal{I}}$, and an *n*-adic predicate denotes a set of members of $\boldsymbol{\mathcal{I}}$; an atomic formula like Pa is true iff $\delta(a) \in \delta(P)$. A notable difference with standard semantics for first-order languages is the fact that the denotation of an *n*-adic predicate can correspond to a set belonging to $\boldsymbol{\mathcal{R}}_n$, that is a set of *n*-tuples of individuals resembling each other in some aspect. Those predicates can be characterized as *real* predicates, predicates corresponding to real properties or relations. This feature will be very important in a weak nominalist interpretation of second-order languages that we will see later.

One last remark concerning those two kinds of nominalism I have presented so far: they cannot struggle against the well-known problems of extensionality illustrated by the example of “cordate” and “renate”; in order to deal with this, we could refine the framework in various way, for example by using Lewis’ possible worlds strategy. But I wish to keep the frameworks as simple as possible and thus I will not follow this way here.

2.3. Weak realist framework

I take weak realism as the view according to which the world is made out of states of affairs consisting of the instantiation of an *n*-adic universal by *n* individuals. Therefore, there are universals and individuals but every universal is instantiated and every individual instantiates at least one universal.

A weak realist framework could be defined as a structure $<\boldsymbol{\mathcal{S}}, \boldsymbol{\mathcal{U}}, \boldsymbol{\mathcal{I}}>$ where $\boldsymbol{\mathcal{I}}$ is a set of urelements (intuitively the individuals), $\boldsymbol{\mathcal{U}}$ is a set of this form $\{\boldsymbol{\mathcal{U}}_1, \ldots \boldsymbol{\mathcal{U}}_n, \ldots\}$ such that each member of $\boldsymbol{\mathcal{U}}$ is disjoint of each other and of $\boldsymbol{\mathcal{I}}$ (intuitively the $\boldsymbol{\mathcal{U}}_n$’s are sets of *n*-adic universals), and $\boldsymbol{\mathcal{S}}$ is a set of couples whose first term is a member of $\boldsymbol{\mathcal{U}}_n$ and the second term is an *n*-tuple of members of $\boldsymbol{\mathcal{I}}$ (intuitively it is an *n*-adic state of affairs), and such that every member of the $\boldsymbol{\mathcal{U}}_n$’s and of $\boldsymbol{\mathcal{I}}$ appears in at least one member of $\boldsymbol{\mathcal{S}}$. (This condition expresses the fact that every universal and every individual appear in at least one state of affairs.)

But $\boldsymbol{\mathcal{U}}$ and $\boldsymbol{\mathcal{I}}$ are not basic sets (*i.e.* sets whose constructions cannot be entirely determined by another element of the framework): $\boldsymbol{\mathcal{U}}$ and $\boldsymbol{\mathcal{I}}$ are indeed entirely determined by $\boldsymbol{\mathcal{S}}$. Thus I could equivalently define the weak realist framework as a structure $<\boldsymbol{\mathcal{S}}>$ where $\boldsymbol{\mathcal{S}}$ is a set of couples satisfying the following conditions:

i) The first term of every couple of $\boldsymbol{\mathcal{S}}$ is an urelement. Let us call those urelements *universals.*

ii) The second term of every couples of $\boldsymbol{\mathcal{S}}$ is a tuple of urelements. Let us call those urelements *individuals.*

iii) The set of individuals is disjoint from the set of universals.

iv) If a universal X is the first term of a couple of $\boldsymbol{\mathcal{S}}$ whose second term is an n-tuple, then every couple of $\boldsymbol{\mathcal{S}}$ whose first term is X has also an n-tuple as second term. (This condition assures that every universal has a defined adicity. One may argue that certain predicates like *surround* do not have a defined adicity, but I will not deal with this kind of case.)

We can now define $\boldsymbol{\mathcal{I}}$ as the set of individuals, the $\boldsymbol{\mathcal{U}}_n$'s as sets of universals appearing in couples of $\boldsymbol{\mathcal{S}}$ whose second members are n-tuples, and $\boldsymbol{\mathcal{U}}$ as the set of $\boldsymbol{\mathcal{U}}_n$'s. The resulting framework is strictly equivalent to the first one.

Why should we prefer this second framework to the first? Because this framework is more conform to the essential idea of weak realism as I defined it: the world is made out of state of affairs uniquely. The universals and individuals are only abstracted from them, they are not what is really fundamental, though they are the entities of the frameworks. In formal terms: $\boldsymbol{\mathcal{U}}$ and $\boldsymbol{\mathcal{I}}$ are not basic sets, though they are indeed sets of urelements.

A model of a weak realist semantics must be a structure: $<\boldsymbol{\mathcal{S}}, \delta>$, where $\boldsymbol{\mathcal{S}}$ (and also $\boldsymbol{\mathcal{U}}$ and $\boldsymbol{\mathcal{I}}$) is defined as earlier and δ is a denotation function. A weak realist semantics for first-order logic is very different from standard semantics (which, as we have seen, is a strong nominalist semantics). The denotation function would map an n-adic predicate to a member of $\boldsymbol{\mathcal{U}}_n$, while in standard semantics the denotation of an n-adic predicate is a subset of $\boldsymbol{\mathcal{I}}^n$ (i.e. the set of every n-uple of members of $\boldsymbol{\mathcal{I}}$). In less formal terms: in the standard semantics a predicate denotes directly its extension while in the weakly realist semantics it denotes a universal.

Let us take a quick look to the semantic rule for an atomic formula Pa. In a standard nominalist model, this formula is true iff $\delta(a) \in \delta(P)$. In a weak realist model, it is more complicated since $\delta(P)$ does not denote directly the extension of P: it denotes a monadic universal, and the formula is true iff there is a state of affairs in which the individual denoted by a instantiates the monadic universal denoted by P. Therefore the rule is: Pa is true iff $<\delta(P), \delta(a)> \in \boldsymbol{\mathcal{S}}$.

A weak realist could of course prefer to use the standard semantics for first-order logic (because it is standard and easier). But if a weak realist does so, that means that *he does not take semantics seriously*: the standard semantics for first-order languages does not provides a picture of the world/language relation as he thinks it is. (From its ontological point of view, the world is a world of state of affairs, not a world of individuals only.) Only a semantics constructed from the weak realist framework can represent adequately the way things are from a weak realist perspective.

Moreover, standard semantics and weak realist semantics for first-order logic are not equivalent. The following is a schema of valid formula in weak realist semantics (since every universal is instantiated), but not in standard one:

$$\exists x_1 \dots x_n \; K^n x_1 \dots x_n \quad \text{for any predicate } K^n$$

2.4. Strong realist framework

According to a strong realist, there are universals, they may be uninstantiated, and individuals only exist as they instantiate universals. Let us assume that this last condition means that every individual instantiates at least one universal.

A strong realist framework may be represented as a structure: $<\mathcal{U}, \mathcal{I}, \varepsilon>$ where $\mathcal{I}$ is a set of urelements (intuitively the individuals), $\mathcal{U}$ is a set $\{\mathcal{U}_1, \ldots \mathcal{U}_n, \ldots\}$ whose members are sets of urelements all disjoint from each other and from $\mathcal{I}$ (intuitively $\mathcal{U}_n$ is the set of n-adic universals), and ε is a function mapping certain members of the $\mathcal{U}_n$'s to a non-empty subset of $\mathcal{I}^n$ (i.e. the set of n-tuples of members of $\mathcal{I}$). Intuitively, ε is an *instantiation function*. Note that it maps *certain members* of the $\mathcal{U}_n$'s, not necessary all of them, thus there may be uninstantiated universals.

Moreover, since every individual must be such that it instantiates at least one universal, I must add the following condition to the model: for every member X of $\mathcal{I}$, there is a member Y of one of the $\mathcal{U}_n$'s such that X is a term of one of the n-tuples member of $\varepsilon(Y)$.

Let us consider functions as sets of couples. This strong realist framework contains thus three sets: $\mathcal{I}$, $\mathcal{U}$ et ε. Are they all *basic sets*?

Given $\mathcal{U}$ and $\mathcal{I}$ it is impossible to deduce ε. For example, in a very simple framework with only the universals *Red* and *Green* and the individual *i*, it is impossible to say if *i* is *Green* or *Red* (or both). All we know is that *i* instantiates at least one of these two universals. The instantiation function is underdetermined by $\mathcal{U}$ and $\mathcal{I}$, therefore it is indeed a basic set. Instantiation is fundamental.

On the contrary, given ε it is possible to deduce $\mathcal{I}$ since I have supposed that every member of $\mathcal{I}$ is instantiated: every member of $\mathcal{I}$ will thus appear somewhere in the mappings of ε. It is thus possible to define $\mathcal{I}$ from the function ε as follows: X is a member of $\mathcal{I}$ iff there is a Y member of $\mathcal{U}_n$ such that $\varepsilon(Y) = <X_1, \ldots, X_n>$ and X is one of the $X_1, \ldots, X_n$.

Can we also deduce $\mathcal{U}$ from ε? No. Consider the simple framework in which $\mathcal{I}$ contains only the individual *i,* and ε only maps the universal *Red* to *i*. Does that mean that $\mathcal{U}$ only contains *Red*? No, for there may be uninstantiated universals: maybe the uninstantiated universal *Green* is another member of $\mathcal{U}$. This set is underdetermined by the instantiation function ε. Therefore $\mathcal{U}$ is a basic set. Universals are fundamental.

We could thus redefine the strong realist framework as a structure $<\mathcal{U}, \varepsilon>$ where $\mathcal{U}$ is a set $\{\mathcal{U}_1, \ldots \mathcal{U}_n, \ldots\}$ whose members are disjoint sets of urelements, and ε is a function mapping certain members of the $\mathcal{U}_n$'s to a non-empty set of n-tuples of urelements all distinct from every members of the $\mathcal{U}_n$'s. The set $\mathcal{I}$ can now be defined as the sets of those urelements. I think this framework reflects more closely the strong realist view according to which individuals do not exist by themselves but only through the instantiation of a universal.

Note that we could define a function ε^* mapping *every* members of the $\mathcal{U}_n$'s to a (*possibly empty*) subset of $\mathcal{I}^n$. (Recall that ε maps *certain* members of the $\mathcal{U}_n$'s to a *non-empty* subset of $\mathcal{I}^n$.) The case where a universal is not instantiated would be represented as the case where ε^* maps this universal to the empty set. Now, we could construct $\mathcal{U}$ from ε^* and thus we could take a structure $\langle\varepsilon^*\rangle$ as an equivalent strong realist framework. In what extent $\mathcal{U}$ is really a basic set?

In fact, each ontological framework can be represented in different equivalent ways. How are we supposed to choose among them? We must remember that we are trying to construct *ontological frameworks*; we must thus construct those frameworks in an ontologically relevant way: it must reflects as closely as possible the ontological claims we are dealing with. For the present matter, those claims are: universals are fundamental entities, they may be uninstantiated, and individuals only exist as they instantiate universals. The structure $\langle\mathcal{U}, \varepsilon\rangle$ seems to be the more adequate, where $\mathcal{U}$ and ε are both basic and ε is a function mapping each *instantiated* universal to its extension. Uninstantiated predicates have no instantiation sets at all. On the contrary, in the structure with ε^*, uninstantiated predicates do have an instantiation set: the empty set. We can observe that an uninstantiated predicate would be somehow coextensive to every other uninstantiated predicate even of a different adicity. It seems to me that it is a less elegant picture of the strong realist ontology; but I must admit that I cannot find any conclusive reason for choosing the framework $\langle\mathcal{U}, \varepsilon\rangle$ rather the other.

Let us compare strong realism and weak realism. They agree on entities: there are two kinds of entities, individuals and universals. But they disagree on what is fundamental: for the weak realist, neither individuals nor universals are fundamental, only state of affairs connecting individuals to universals are fundamental; for the weak realist, universals are fundamental (that means that the set of universals is a basic set), and the relation of instantiation is also fundamental, but the individuals are not fundamental. (Recall that entities are the urelements of the framework, while fundamentality is what characterizes the basic sets of the framework; therefore, "universals are *fundamental entities*" means that the set of urelements standing for universals is a basic set of the framework.)

Let us finally take a quick look to a strong realist semantics for first-order language. A strong realist model for this language should be defined as a structure $\langle\mathcal{U}, \varepsilon, \delta\rangle$ satisfying the following conditions: $\mathcal{U}$ and ε (and also $\mathcal{I}$) are defined as we have seen in this section; δ is a denotation function mapping constant objects to a member of $\mathcal{I}$ and *n*-adic predicates to a member of $\mathcal{U}_n$ if $\mathcal{U}_n$ is not empty, and it is not defined otherwise. The semantic rules for an atomic formula Pa will be: Pa is true iff $\varepsilon(\delta(P))$ is defined and $\delta(a) \in \varepsilon(\delta(P))$; in other terms, Pa is true iff the predicate P denotes a universal that is instantiated by the individual denoted by a.

Does this semantics seem too complicated? I can only repeat what I said about weak realism: a strong realist could of course prefer standard semantics for first-order languages since it is much simpler, but doing so would mean that he refuses to take semantics seriously. Standard semantics does not accurately describe the way first-order sentences are being made true from a strong realist point of view. What makes true that Pa is true, it is not the fact that a is the name of an individual which belongs to the set denoted by P; it is that P is the name of a universal and a is the name of an individual such that this individual instantiates that universal. This is the way it *really* works according to strong realism, and an ontologically relevant semantics must be able to represent it.

2.5. Tropist framework

Since Williams [8], a new beast has appeared (or reappeared under a new name) in the ontological zoo: tropes. They constitute a sort of intermediate entity between (concrete) individuals and (abstract) universals. They can be intuitively described as *abstract particulars*. For example the red of a particular apple is a trope ; according to the tropist view, *the-red-of-this-apple* is a distinct entity, both distinct from this apple and from the universal *Red*.

The aim of a tropist account is to eliminate both universals and individuals by taking only tropes as entities. Concrete individuals will be constructed as bundles of *compresent* tropes, universals as bundles of *resembling* tropes. Compresence is supposed to be an equivalence relation, and resemblance a reflexive and symmetric relation.

The tropist picture of the world is easy to grasp when we consider only tropes of property, i.e. monadic tropes. Thus a monadic tropist framework could be defined as a structure $<\mathcal{T}, \mathcal{C}, \mathcal{R}>$ where $\mathcal{T}$ is a set of urelements (intuitively the tropes), $\mathcal{C}$ is a partition of $\mathcal{T}$ (intuitively, each member of $\mathcal{C}$ is a set of compresent tropes) and $\mathcal{R}$ is a set of non-empty sets of members of $\mathcal{T}$ such that every member of $\mathcal{T}$ appears in at least one member of $\mathcal{R}$ (intuitively, every member of $\mathcal{R}$ is a set of tropes resembling each other in some aspect, and every tropes resembles at least itself).

We note immediately that $\mathcal{T}$ is not a basic set of the framework. Thus, an equivalent framework could be $<\mathcal{C}, \mathcal{R}>$ where $\mathcal{C}$ is a set of non-empty disjoint sets of urelement, we defined the set $\mathcal{T}$ as the union of the members of $\mathcal{C}$, and $\mathcal{R}$ is defined as before. Thus, tropes are the only entities; and what is fundamental is the compresence and resemblance structures.

This framework allows us to interpret first-order languages with only monadic predicates, roughly in the following way: a model is a structure $<\mathcal{C}, \mathcal{R}, \delta>$; a constant object denotes a member of $\mathcal{C}$; a monadic predicate denotes a member of $\mathcal{R}$; an atomic formula like Pa is true in a model iff $\delta(P)$ and $\delta(a)$ have a common member. Intuitively, that means that an individual is a set of compresent tropes, for example an apple is a set of tropes among which we find *the-red-of-this-apple*; a universal is a set of resembling tropes, for example the universal *Red* is a set of tropes resembling each other like the *red-of-this-apple*, *the-red-of-this-tomato* and so forth; it is true that the apple is red because the trope *the-red-of-this-apple* belongs to the set of compresent tropes constituting the apple, and because *the-red-of-this-apple* belongs also to the set of resembling tropes constituting the universal *Red*.

Now, let us introduce the idea of relational tropes. There is no consensus about how to do it. For example Bacon [9], Mertz [10] and Schneider [11] defend (in different ways) that relational tropes are not reducible to non-relational one, while Campbell [12] defends the opposite view. Here, I will follow a variant of this last view. I assume that an n-adic relational tropes is in fact n ordered monadic tropes T_1, ..., T_n. What makes those tropes relational is the way they resemble other tropes: the ordered set of tropes T_1, ..., T_n resembles other ordered sets of n tropes (while genuine monadic tropes only resemble each other individually). For example, the relational trope of *the-love-of-Romeo-for-Juliet* is in fact a couple of monadic tropes: roughly speaking, the trope of *the-love-for-Juliet* (a trope belonging to the cluster of compresent tropes constituting Romeo) and the trope of *the-being-loved-by-Romeo* (a trope belonging to the cluster of compresent tropes constituting Juliet). This couple of tropes resembles the couple of tropes formed by *the-love-for-Desdemona* and *the-being-loved-by-Othello*. The universal *Love* is the set of every such couple of tropes resembling each other. (Of course, this formulation is not completely adequate, but the idea is, I think, easy to grasp.)

I will now define my tropist framework as a structure $<\boldsymbol{\mathcal{C}}, \boldsymbol{\mathcal{R}}>$ satisfying the following condition. First, $\boldsymbol{\mathcal{C}}$ is (as previously) a set of non-empty disjoint sets of urelements (intuitively it is the compresence structure of the world). Then we define the set $\boldsymbol{\mathcal{T}}$ as the union of the members of $\boldsymbol{\mathcal{C}}$; let us call them *tropes*. Finally, let us define $\boldsymbol{\mathcal{R}}$. Intuitively, it will be a set whose members are sets of resembling tropes; since relational tropes are resembling each other only when they are taken in ordered sets, $\boldsymbol{\mathcal{R}}$ will be a set whose members are sets of n-tuples of tropes. (We admit that the 1-tuple $<X>$ is simply X.) The set $\boldsymbol{\mathcal{R}}$ must also satisfy two conditions:

i) The first condition expresses the fact that each tropes is either a genuine monadic trope, or it is a trope forming an n-adic relational trope with $n - 1$ other tropes. The condition is: if a trope T appears in an n-tuple $<T_1, ..., T, ..., T_{n-1}>$ belonging to a member of $\boldsymbol{\mathcal{R}}$, then there is no other n-tuple belonging to a member of $\boldsymbol{\mathcal{R}}$ in which T appears.

ii) The second condition expresses the fact that each trope resembles at least itself: each trope appears in an n-tuple belonging to at least one member of $\boldsymbol{\mathcal{R}}$.

Now we can complete our sketch of a tropist semantics for first-order languages: as before, a constant object denotes a member of $\boldsymbol{\mathcal{C}}$; an n-adic predicate denotes a member of $\boldsymbol{\mathcal{R}}$ whose members are n-tuples; and the atomic formula Rab is true in a model iff one of the couples of tropes denoted by R is such that the first term is a trope belonging to the set of tropes denoted by a and the second term is a trope belonging to the set of tropes denoted by b. More briefly: Rab is true iff there are X and Y such that $<X, Y> \in \delta(R)$ and $X \in \delta(a)$ and $Y \in \delta(b)$.

There is no standard approach in tropes theory, especially concerning the question of relational tropes, and I cannot deal with every variant; hence this tropist framework is only one among many others that could be constructed, but I think it is a convincing tropist account of reality.

3. Semantics for second-order languages

In the rest of this paper I will take a closer look to semantics for second-order languages. In addition to individual variables we will have n-adic predicate variables. We expect this second-order quantification to express quantification over properties and relations, in such a way that for example this formula:

(LL) $\forall F(Fx \equiv Fy) \rightarrow x = y$

expresses Leibniz' identity of indiscernibles: if two objects *have exactly the same properties* then they are identical.

It is obvious that such interpretation of second-order quantification may come very useful to metaphysicians. Important metaphysical laws such as Leibniz Laws cannot be expressed without using it. However, because of Quine's criterion for ontological commitment, it is commonly thought that second-order quantification automatically brings ontological commitment to some sort of universals. But with the truthmaking criterion for ontological commitment, I will show that this view is not correct: we can quantify over properties and relations in a nominalist framework.

I will construct a semantics for second-order languages within each ontological framework I have defined in section 2: strong and weak nominalism, weak and strong realism, and tropism. And I will show that the different semantics are not equivalent; the disagreement between those five ontological views extend from ontology to logic.

3.01. Definition of second-order languages with identity

A vocabulary $\boldsymbol{\mathcal{V}}$ is a set of non-logical constants: it contains constant objects (noted for example a, b, c, etc.) and n-adic constant predicates (noted for example P^n, Q^n, R^n, etc.) with an integer $n \geq 1$. Assuming a set of variable objects $\{x_1, x_2, \ldots\}$ and sets of n-adic variable predicates $\{F_1{}^n, F_2{}^n, \ldots\}$ for each adicity, a second-order language $\boldsymbol{\mathcal{L}_{\mathcal{V}}}$ is a set of formulas constructed in accordance with the following rules:

(i) *Atomic formulas.* If K^n is an n-adic term predicate (i.e. a constant predicate of $\boldsymbol{\mathcal{V}}$ or a variable predicate) and $t_1, \ldots, t_n$ are n term objects (i.e. a constant object of $\boldsymbol{\mathcal{V}}$, or a variable object), then this is a formula of $\boldsymbol{\mathcal{L}_{\mathcal{V}}}$: $K^n(t_1, \ldots, t_n)$.

(ii) *Identity formula.* If α and β are both term objects or both *n*-adic term predicates of the same adicity, then this a formula of $\mathcal{L}_V$: α = β.

(iii) *Quantified formulas.* If φ is a formula of $\mathcal{L}_V$ and V is a variable (a variable object or a variable predicate), then these are formulas of $\mathcal{L}_V$: $\forall V(\varphi)$, $\exists V(\varphi)$.

(iv) *Complex formulas.* If φ and ψ are formulas of $\mathcal{L}_V$, these are formulas of $\mathcal{L}_V$: $(\neg\varphi)$, $(\varphi \,\&\, \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$ and $(\varphi \equiv \psi)$.

(v) *Closure.* Every formula of $\mathcal{L}_V$ is constructed by a finite number of applications of the preceding rules.

Note that (i) allows variable predicates and objects to occur freely in $\mathcal{L}_V$.

Wherever there is no ambiguity, I will drop parenthesis and *n*'s for adicity, and I will usually write $K^n t_1 \ldots t_n$ instead of $K^n(t_1, \ldots, t_n)$. Note also that I will use *x*, *y* and *z* as variable objects instead of the official x_1, x_2, etc., and F^n, G^n and H^n, instead of the official $F_1{}^n$, $F_2{}^n$, etc..

I will generally use *t* as metavariable for term object, *c* for constant object, *v* for variable object, K^n for *n*-adic term predicate (constant or variable), C^n for *n*-adic constant predicate, and V^n for *n*-adic variable predicate., and φ and ψ for formulas. (Wherever I make a different use of those metavariables, the changes will be carefully specified.)

3.1. Strong nominalist second-order logic

3.1.1. A strong nominalist semantics: SN_{SOL}

How can we interpret second-order languages in a strong nominalist framework? A model in a strong nominalist semantics will be simply a structure $<\mathcal{I}, \delta>$ where $\mathcal{I}$ is a set of urelements and δ a denotation function. Since there is nothing but an unstructured set of individuals, it seems difficult at first glance to make sense of second-order quantification as *quantification over properties and relations*.

The most natural solution is to consider that we quantify somehow over the constant predicates themselves. (We will also see in 3.1.4. another solution, which I think is unsatisfying.)

A very simple and elegant solution would be to interpret quantification over predicates as substitutional quantification. Indeed, substitutional quantification is a way to quantify over the constants of the language. A problem with this account is that it is difficult to make sense of formulas with free variable predicates. (Maybe it would require to modify the language in order to prevent predicate variables to occur freely.) I prefer nevertheless to stick with objectual quantification which is, I think, a more intuitive way to understand quantification. I will show that we can construct an appropriate domain of quantification for variable predicates of a second-order language $\mathcal{L}_{\mathcal{V}}$, using only the domain of individuals $\mathcal{I}$, the vocabulary $\mathcal{V}$, and the denotation function δ.

Another preliminary remark: a nominalist should not be inclined to think that every constant object denotes an individual. Indeed, it would mean that names carry an ontological import, and I see no reason why a nominalist would endorse this view (and I will not see neither why a realist or a tropist would accept it): the world is independent from the language, and thus we cannot expect every element of our language to correspond to something in the world. Therefore, if we want our semantics to represent nominalism accurately, we should not presume that every constant object denotes an individual. Hence the resulting logic will be a free logic.

Now, let us define a strong nominalist semantics for second-order languages.

A nominalist model for a second-order language $\mathcal{L}_{\mathcal{V}}$ is a structure $<\mathcal{I}, \delta>$ where $\mathcal{I}$ is a non-empty set of urelements (intuitively the individuals) and δ is a denotation function mapping certain constant objects of $\mathcal{V}$ to a member of $\mathcal{I}$ and every *n*-adic constant predicate of $\mathcal{V}$ to a (possibly empty) subset of $\mathcal{I}^n$ (the set of every *n*-tuple of members of $\mathcal{I}$).

Intuitively that is to say: a constant object may denote an individual (or not), and each *n*-adic constant predicate denotes a set of *n*-tuples of individuals, which can be seen as the *extension* of the predicate (and this extension set can be empty).

Let us assume that the 1-tuple $<X>$ is identical to X. Hence the extension of a monadic predicate is a (possibly empty) set of individuals.

How can we deal with variable predicates in this framework? As I suggested earlier, let us assume that second-order quantification in this nominalist framework is quantification over the constant predicates. Since constant predicates denote their extensions, the range of an *n*-adic variable predicate should be *the set of the extensions of every n-adic constant predicate of the vocabulary.* For example, if there are only two monadic predicates P^1 and Q^1 in $\mathcal{V}$, then the range of the variable F^1 contains only the extension of P^1 and the extension of Q^1. That is the general idea. Let us now give to this idea a rigorous formulation.

We need to describe more carefully the structure of the set $\mathcal{V}$, the vocabulary of our second-order language $\mathcal{L}_{\mathcal{V}}$. So far, all I have assumed is that it contains constant objects and constant predicates. I will assume now that $\mathcal{V}$ is constructed from sets of constants of different types: suppose that $\mathcal{V}_0$ is a set of constant objects, $\mathcal{V}_1$ is a set of monadic predicates, and more generally $\mathcal{V}_n$ is a set of *n*-adic predicates ($n \geq 1$). A vocabulary $\mathcal{V}$ is the union of these sets. For an integer $k \geq 1$ such that $\mathcal{V}$ does not contain any *k*-adic predicate, $\mathcal{V}_k$ is empty.

For every $n \geq 1$, I define the set $\mathcal{P}_n$ as follows:

If $\mathcal{V}_n$ is non empty, then $\mathcal{P}_n = \{\delta(X) : X \in \mathcal{V}_n\}$

If $\mathcal{V}_n$ is empty then $\mathcal{P}_n$ is empty too.

Intuitively, $\mathcal{P}_n$ is the set of the extensions of *n*-adic predicate of $\mathcal{V}$. And if there are no *n*-adic predicates, then the set of their extensions is the empty set.

If there is at least one monadic predicate in $\boldsymbol{\mathcal{V}}$, then $\boldsymbol{\mathcal{P}}_1$ is a set whose members are extensions of monadic properties, i.e. subsets of the domain of individuals $\boldsymbol{\mathcal{I}}$. Otherwise, if there are no monadic predicates in $\boldsymbol{\mathcal{V}}$, then $\boldsymbol{\mathcal{P}}_1$ is the empty set.

Thus we have: $\boldsymbol{\mathcal{P}}_1 \in \mathfrak{P}(\boldsymbol{\mathcal{I}})$ (i.e. the power set of $\boldsymbol{\mathcal{I}}$). More generally, it is easy to prove that $\boldsymbol{\mathcal{P}}_n \in \mathfrak{P}(\boldsymbol{\mathcal{I}}^n)$. Every $\boldsymbol{\mathcal{P}}_n$ is a (possibly empty) set of n-tuples of individuals.

It is very important to note that the sets $\boldsymbol{\mathcal{P}}_n$'s do not add anything to our ontology: their construction is entirely determined by the set of individuals $\boldsymbol{\mathcal{I}}$, the set of n-adic predicates $\boldsymbol{\mathcal{V}}_n$ and the denotation function δ.

We can now define a value-assignment, say more simply an *assignment*, for variable predicates as well as for variable objects.

A function s is an *assignment* if it satisfies those two conditions:

i) For every variable object v, $s(v) \in \boldsymbol{\mathcal{I}}$.

ii) For every n-adic variable predicate V^n, if $\boldsymbol{\mathcal{P}}_n$ is non empty then $s(V^n) \in \boldsymbol{\mathcal{P}}_n$, and if $\boldsymbol{\mathcal{P}}_n$ is empty then $s(V^n)$ is not defined.

In other terms, a value-assignment maps each variable object to an individual (that is standard), and it maps each n-adic variable predicate to the extension of an n-adic constant predicate if the vocabulary contains at least one such predicate, otherwise the variable predicate has no assignment.

We define a *denotation function under a given assignment.* The denotation under the assignment s of any term τ (either term object or term predicate), is the function δ_s such that $\delta_s = \delta(\tau)$ if τ is a constant (object or predicate), and $\delta_s = \mathrm{s}(\tau)$ if τ is a variable (object or predicate). Intuitively that means that assignment plays the role of denotation function for variables.

We can now define recursively the notion of *truth in a model under an assignment* for every formula of $\boldsymbol{\mathcal{L}_\mathcal{V}}$. We note $Ms \vDash \varphi$ to express that the formula φ is true in the model M under the assignment s.

(i) *Atomic formulas.*

$Ms \vDash K^n t_1 ... t_n$ iff $\delta_s(t_1), ..., \delta_s(t_n)$ and $\delta_s(K^n)$ are all defined and $<\delta_s(t_1), ..., \delta_s(t_n)> \in \delta_s(K^n)$

(ii) *Identity formulas.*

$Ms \vDash \alpha = \beta$ iff $\delta_s(\alpha)$ and $\delta_s(\beta)$ are both defined and are the same.

(iii) *Quantified formulas.*

$Ms \vDash \forall v(\varphi)$ iff $Ms' \vDash \varphi$ for every assignment s' agreeing with s except possibly on v.
$Ms \vDash \exists v(\varphi)$ iff $Ms' \vDash \varphi$ for an assignment s' agreeing with s except possibly on v.
$Ms \vDash \forall V^n(\varphi)$ iff $Ms' \vDash \varphi$ for every assignment s' agreeing with s except possibly on V^n.
$Ms \vDash \exists V^n(\varphi)$ iff $Ms' \vDash \varphi$ for an assignment s' agreeing with s except possibly on V^n.

I skip the rules for complex formulas which are standard. The notions of truth in a model, satisfiability and validity are also defined as usual. Let us call the theory so defined SN_{SOL}.

3.1.2. Remarkable features of SN_{SOL}

According to (ii), the identity $a = a$ is not true iff a does not denote an individual. The same goes for non-denoting variable predicates: F^n is a non-denoting variable iff there are no n-adic constant predicates in the vocabulary $\boldsymbol{\mathcal{V}}$, and in such case $F^n = F^n$ is not true. Therefore this formula:

(ID) $\exists F^n(F^n = F^n)$

is false iff there are no n-adic predicates in the vocabulary. In other terms, (ID) is valid for every adicity *iff the vocabulary contains constant predicates of every adicity*. In other terms, the formula $\exists F^n(F^n = F^n)$ in SN_{SOL} expresses the fact that there is at least one n-adic constant predicate.

It would be incorrect to say that (ID) is not valid: its validity depends on the language we are interpreting. In models for second-order languages containing predicates of every adicity, (ID) *is* valid. We can say that (ID) is not *generally valid*, if by that expression we mean that it is not valid *for every second-order language*. (ID) is only *occasionnally valid* in SN_{SOL}. (Usually, this distinction is useless since valid formulas are generally valid; this is a special case. In the rest of this paper, *valid* without further specification means *generally valid*.)

We may notice that the equivalence $\forall F^n(\varphi) \equiv \neg\exists F^n\neg(\varphi)$ does not hold if there are no n-adic predicates in $\boldsymbol{\mathcal{V}}$. Suppose indeed that φ is $F^n x_1 ... x_n$: then $\forall F^n(\varphi)$ is false and $\neg\exists F^n\neg(\varphi)$ is true.

SN_{SOL} is a free logic for constant objects: certain constant objects may not denote any individual. Therefore this formula schema is not valid:

(C-IND) $\exists x(x = c)$ where c is a constant object.

Indeed, this formula is false in a model where c does not denote any individual. On the other hand, the logic is not free for constant predicates. The following formula schema is valid:

(C-PRED) $\exists F^n(F^n = C^n)$ where C^n is an n-adic constant predicate

It is an expected result in our strong nominalist semantics since quantification with a variable predicate is precisely understood as a sort of quantification over the constant predicates of the same adicity.

An interesting feature of SN_{SOL} is that two coextensive predicates are identical. It seems to be an expected consequence of the strong nominalist framework, which is purely extensional: since a predicate denotes its extension directly, and since two predicates are identical if they denote the same thing, then two predicates having the same extension are expected to be identical.

We can express this idea by this formula:

(COEXT) $\forall x_1 ... \forall x_n(F^n x_1 ... x_n \equiv G^n x_1 ... x_n) \equiv F^n = G^n$

But this formula can be false in SN_{SOL}. Suppose that there are no n-adic predicates: $F^n x_1 ... x_n$ and $G^n x_1 ... x_n$ are both false (since F^n and G^n lack denotation) and thus $F^n x_1 ... x_n \equiv G^n x_1 ... x_n$ is true; on the other hand, $F^n = G^n$ is false (since F^n and G^n lack denotation). So, like (ID), the formula (COEXT) is not *generally* valid but it is *occasionally* valid: it is valid in models for languages containing predicates of every adicity.

But if we add to the formula the condition $\exists F^n(F^n = F^n)$ which expresses the fact that there is at least one n-adic constant predicate, then we obtain a generally valid formula:

(ID+COEXT) $\exists F^n(F^n = F^n) \rightarrow (\forall x_1 ... \forall x_n(F^n x_1 ... x_n \equiv G^n x_1 ... x_n) \equiv F^n = G^n)$

Another interesting feature of this semantics is that there may be *unsatisfied constant predicates.* i.e. constant predicates whose extensions are empty. We could express this idea with this formula:

(UNINST) $\exists F^n(F^n = F^n) \ \& \ \exists F^n \forall x_1 ... \forall x_n \neg F^n x_1 ... x_n$

This formula is indeed true iff there is at least one n-adic constant predicate and no n-tuple of individuals satisfies this predicate. This formula in SN_{SOL} is not valid but it is satisfiable, i.e. it is true in some models but not all.

A last interesting feature of the semantics is that there may be *bare individuals*, i.e. individuals which do not belong to the extension of any predicate. This idea is expressed by the following formula:

(BARE) $\exists x \forall F^n \forall y_1 ... \forall y_{(n-1)}(\neg F^n(x, y_1, ..., y_{(n-1)}) \ \& \ ... \ \& \ \neg F^n(y_1, ..., y_{(n-1)}, x))$

This formula is not valid but it is satisfiable in SN_{SOL}.

It is interesting at this point to see how each ontological thesis is somehow reflected by the *logical status* of a certain formula. The identity of coextensive predicate is expressed by the validity of (ID+COEXT); the possibility of unsatisfied predicates is expressed by the satisfiability of (UNINST); and the possibility of bare individuals is expressed by the satisfiability of (BARE). And since the logical statuses of those formula are just a consequence of the strong nominalist model, we can now justify very rigorously why strong nominalism implies those ontological thesis.

3.1.3. Other ontological claims. Variants of SN_{SOL}

Suppose now that a strong nominalist wants to make further requirements on the frameworks besides the fact that there are individuals and nothing more. For example:

(1) Every constant predicate is satisfied.
(2) There are properties (monadic predicates).

(3) Every object have at least one property.

(4) Two objects having exactly the same properties are identical.

Can we represent those claims in the semantics?

For (1), we have to impose to δ the following condition: if C^n is an n-adic constant predicate of $\boldsymbol{\mathcal{V}}$, then $\delta(C^n)$ is *a non-empty* subset of $\boldsymbol{\mathcal{I}}^n$. (The original condition did not prevent $\delta(C^n)$ from being empty.) In such a case, (UNINST) is no longer satisfiable: its negation becomes a valid formula.

For (2), we simply have to impose that $\boldsymbol{\mathcal{V}}_1$ is not empty.

For (3), we have to impose that a model of a language $\boldsymbol{\mathcal{L}_\mathcal{V}}$ is such that for every X member of $\boldsymbol{\mathcal{I}}$ there is a Y member of $\boldsymbol{\mathcal{V}}_1$ such that $X \in \delta(Y)$. In such a case, (BARE) is no longer satisfiable; its negation becomes a valid formula.

For (4), we have to impose that a model of a language $\boldsymbol{\mathcal{L}_\mathcal{V}}$ is such that two distinct members of $\boldsymbol{\mathcal{I}}$ do not belong to exactly the same extensions of members of $\boldsymbol{\mathcal{V}}_1$. (More rigorously: if X_1 and X_2 are two distinct members of $\boldsymbol{\mathcal{I}}$ then there must be at least one member Y of $\boldsymbol{\mathcal{V}}_1$ such that $\delta(Y)$ contains X_1 and not X_2, or X_2 and not X_1.) Under this condition, the formula often quoted as Leibniz Law would be valid (generally valid):

(LL) $\forall F(Fx \equiv Fy) \rightarrow x = y$

Without this condition, this formula is satisfiable but not valid.

There is an interesting feature of claims (1)-(3) from a strong nominalist point of view: *they are not ontological claims* strictly speaking. Indeed, they impose constraints on the language and its interpretation, not on the ontological framework. More generally we can observe that in a strong nominalist framework, a claim about quantification on predicates will not carry ontological significance since predicates are not standing for anything real.

It is worth noting in conclusion that, in this strong nominalist semantics, second-order quantification is interpreted without any ontological commitment to such entities as universals: a model of $\boldsymbol{\mathcal{L}_\mathcal{V}}$ is still simply a structure $<\boldsymbol{\mathcal{I}}, \delta>$, in other terms it contains nothing but a set of individuals and a denotation function; therefore what *makes true* any formula of $\boldsymbol{\mathcal{L}_\mathcal{V}}$ is nothing but the individuals and the way we interpret the language: we have seen how the domain of quantification of each n-adic variable predicate is built from the sets of individuals, the set of relevant constant predicates of the language and the denotation function. Thus, second-order quantification is interpreted in a satisfying way (since it expresses a quantification over properties and relations in a relevant way) and it does not break the strong nominalist requirement: there are individuals and nothing more.

3.1.4. Another strong nominalist second-order logic: SN_{SOL}*

Second-order quantification in a strong nominalist framework could be understood in a different way. While in SN_{SOL} the range of the variable F^n is the set of the extensions of every n-adic constant predicate which *actually* belongs to the vocabulary of our language (thus we quantify somehow over the actual constant predicates of our language), what if the range of the variable F^n was the set of *any possible* extensions of an n-adic constant predicate?

From a strong nominalist point of view, any set of n-tuples of individuals may constitute the extension of a predicate. Therefore, the range of the variable F^n would be simply the power set of $\mathcal{I}^n$. In fact, we will obtain a semantics for second-order languages commonly known as *standard semantics*. Let us call it SN_{SOL}*. (In fact, as I will define it, SN_{SOL}* is not exactly like standard semantics since I will admit non-denoting constant objects, but that is the only difference.)

A model of SN_{SOL}* is a structure $<\mathcal{I}, \delta>$ where $\mathcal{I}$ and δ are defined exactly like in SN_{SOL}. The only difference between SN_{SOL} and SN_{SOL}* appears in the definition of assignment for variable predicates: a function s is an assignment in SN_{SOL}* if s maps every variable object to a member of $\mathcal{I}$ (that is the same as before) and s maps every n-adic variable predicate to a member of $\mathfrak{P}(\mathcal{I}^n)$, i.e. the power set of the set of every n-tuples of members of $\mathcal{I}$. The rest of the semantics is left unchanged.

3.1.5. Remarkable features of SN_{SOL}*

SN_{SOL}* is very different from SN_{SOL}. Let us see the logical statuses of (ID), (COEXT), (UNINST), (BARE) and also (LL).

In SN_{SOL}*, every variable predicate has a denotation (the variable F^n denotes any set of n-tuples of individuals), thus we have the general validity of (ID). (I precise “general” since (ID) is not *generally* valid in SN_{SOL}, it is only valid if the vocabulary of the language contains predicates of every adicity.)

(COEXT) is also generally valid in SN_{SOL}* (while in SN_{SOL} it is only occasionally valid), and *a fortiori* (ID+COEXT) also is generally valid. Like in SN_{SOL}, predicates denotes directly their extensions, therefore two coextensive predicates are identical.

The empty set belongs to the range of every n-adic variable predicate, and thus (UNINST) is not only satisfiable: it is valid in SN_{SOL}*. (In SN_{SOL} it is only satisfiable.)

The set containing all individuals belongs to the range of the variable F, thus (BARE) is not satisfiable; its negation is valid. (In SN_{SOL} (BARE) was satisfiable.)

Finally, the most noticeable fact is the validity of (LL):

$$\text{(LL)} \quad \forall F(Fx \equiv Fy) \rightarrow x = y$$

3.1.6. Is SN_{SOL}* ontologically relevant?

But what does the formula (LL) *mean* in SN_{SOL}*? This formula is supposed to express Leibniz’ identity of indiscernibles: if two individuals have exactly the same properties, then they are identical. The problem with SN_{SOL}* is that the formula (LL) rather expresses the trivial fact that if two individuals *belong to exactly the same sets of individuals*, then they are identical. (Indeed, the range of the variable F is the power set of the set of individuals.)

As I said in the beginning of this section, we expect second-order quantification to express somehow *quantification over properties and relations*: that is why we expect (LL) to express Leibniz Law. Is SN_{SOL}* able to represent such a quantification from a strong nominalist point of view? Well, it is true from a strong nominalist point of view that *any set of individuals may be the extension of a possible monadic predicate*, so a strong nominalist may argue that SN_{SOL}* expresses a certain form of quantification over possible properties and relations. But I think that even a strong nominalist would admit that this interpretation is very dubious. When a strong nominalist is talking about *every property of this apple*, what is (s)he intuitively talking about? Is (s)he talking about every set of individuals containing this apple as SN_{SOL}* would suggest? If (s)he thinks so, (s)he should indeed consider seriously this semantics. But I guess (s)he is rather talking about every property terms we can actually use for the description of this apple; and that is the interpretation expressed by SN_{SOL}. This latter interpretation seems more natural and satisfying.

In conclusion: literally speaking, second-order quantification in SN_{SOL}* is only quantification over sets of n-tuples of individuals. Therefore, though this semantics (which is standard semantics for second-order languages) produces the most expressive second-order logic (it is a well-known fact that no deductive system can be complete for standard second-order semantics), it seems that SN_{SOL}* is ontologically irrelevant.

3.2. Weak nominalist second-order logic

3.2.1. A first weak nominalist semantics: WN_{SOL}

As for strong nominalism, quantification over properties and relations can be understood in two different ways from a weak nominalist point of view. Therefore we will have two weak nominalist semantics: I will call the first WN_{SOL} and the other WN_{SOL}* as they correspond somehow to SN_{SOL} and SN_{SOL}*.

Let us start with WN_{SOL}. I will not say much about it since it is extremely similar to SN_{SOL}. The basic idea is the same: we understand second-order quantification as quantification over the extensions of the constant predicates of the language. WN_{SOL} and SN_{SOL} will differ only as they are constructed from different ontological frameworks, but the resulting logic will be the same.

A model of WN_{SOL} for a language $\mathcal{L}_{\mathcal{V}}$ is a structure $<\mathcal{R}, \delta>$ where $\mathcal{R}$ is defined as in 2.2 (therefore the set $\mathcal{I}$ of individuals is also defined), and δ is a denotation function mapping certain constant objects of $\mathcal{V}$ to a member of $\mathcal{I}$, and every n-adic constant predicate of $\mathcal{V}$ to a (possibly empty) subset of $\mathcal{I}^n$. The rest of the semantics is the same as for WN_{SOL}. An important difference between SN_{SOL} and WN_{SOL} is the fact that in WN_{SOL} an n-adic constant predicate may denote a member of $\mathcal{R}_n$ (i.e. a set of resembling n-tuples of individuals): we can say of this predicate that it is a *real* predicate since it corresponds to an ontological structure. (Of course, we cannot expect every predicate in our language to be a real predicate.)

However, those real predicates do not play any special role in this semantics. We can draw about WN_{SOL} exactly the same conclusion we drew about SN_{SOL}: (ID) is occasionally valid, (C-PRED) is valid, (COEXT) is occasionally valid, (ID+COEXT) is generally valid, and (UNINST) and (BARE) are not valid but satisfiable. Also we can make the same remarks we made in 3.1.3 about the way we should modify the semantics in order to satisfy the ontological claims (1)-(4).

3.2.2. Another weak nominalist semantics: WN_{SOL}*

If we suppose now that the range of a variable predicate F^n is the set of *any possible* extensions of an n-adic constant predicate, as we did previously for SN_{SOL}*, it would also lead us to a semantics very similar to SN_{SOL}*. But we can do something slightly different: let us say that the range of a variable predicate F^n is the set of any possible extensions of a *real* n-adic constant predicate. By *real predicate* I mean a predicate whose extension is a set of resembling n-tuples of individuals. Now, it will not imply that *any* set of n-tuples of individuals can be a value of F^n; only a member of $\mathcal{R}^n$ (a resemblance set) will be a possible value of F^n. If we follow this direction, WN_{SOL}* will be very different from SN_{SOL}*. And I will show that is an interesting theory from an ontological point of view.

The only difference between WN_{SOL} and WN_{SOL}* appears in the definition of assignment for variable predicates: a function s is an assignment in WN_{SOL}* if s maps every variable object to a member of $\mathcal{I}$ (that is the same as before) and every n-adic variable predicate to a member of $\mathcal{R}_n$ (that is the important part). The rest of the semantics is all the same.

3.2.3. Remarkable features of WN_{SOL}*

Let us recall first that the $\mathcal{R}_n$'s are sets of sets of n-tuples of members of $\mathcal{I}$; intuitively those are sets of resembling n-tuples of individual. For example $\mathcal{R}_1$ may contain a set of individuals resembling each other in their redness, etc.. Note that a $\mathcal{R}_n$ may also contain the empty set. It will serve as extension for unsatisfied predicates.

Since an assignment maps *every* n-adic variable predicate to a ressemblance set, (ID) is generally valid in WN_{SOL}*:

(ID) $\exists F^n(F^n = F^n)$

In ontological terms, this formula means that for every adicity n there is at least one set of resembling n-tuples, and thus one possible n-adic real predicate. Since resemblance is reflexive, the validity of (ID) is not a surprise.

One of the most interesting features of WN_{SOL}* concern this formula schema:

(C-PRED) $\exists F^n(F^n = C^n)$ where C^n is an n-adic constant predicate

We have seen earlier that this schema is valid in SN_{SOL} and WN_{SOL} since second-order quantification is somehow quantification over the constant predicates. Is it also valid in WN_{SOL}*? Suppose that the monadic predicate P is not a *real* predicate, i.e. the extension of P is not a set of resembling individuals. In WN_{SOL}*, the variable F must denote a set of resembling individuals; therefore there is no assignment such that F and P denote the same extension. Hence $F = P$ is false for every assignment, and thus $\exists F(F = P)$ is false. It follows that (C-PRED) is not valid. However, it is satisfiable: indeed, (C-PRED) is true iff every constant predicate in the model is a real predicate.

Second-order quantification in WN_{SOL}* is a way to talk about real predicates. For example, the formula $\neg\exists F(F = P)$ is a way to say that P is not a real monadic predicate, and of course it can be true. It is worth noting that even if a predicate is not real, it still has a denotation and thus it may appear in true atomic formulas. The formula Pa & $\neg\exists F(F = P)$ may be true in a model. Therefore, existential generalization for predicate will fail in WN_{SOL}*: Pa does not entail $\exists F(Fa)$. If we translate this in ontological terms, it means: the fact that a satisfies P does not entail that a satisfies a real predicate. That seems correct. The existential generalization only works if you know that P is a real predicate; indeed Pa & $\exists F(F = P)$ entails $\exists F(Fa)$. One may note that predicates in this semantics behave like free terms in free logic (though predicates always denote an extension in this semantics).

Let us see other features of WN_{SOL}*. (COEXT) is generally valid (and *a fortiori* (COEXT+ID) is also generally valid): predicates denote directly their extension, thus two coextensive predicates are identical. (UNINST) is not valid but it is satisfiable: there may be unsatisfied predicates.

(BARE) is not satisfiable. It means that any n-tuple of individuals satisfies at least one possible real predicate, i.e. it belongs to at least one resemblance set. Indeed, since resemblance is reflexive, the $\boldsymbol{\mathcal{R}}_n$ are constructed in such a way that each n-tuple of individuals appears in at least one member of $\boldsymbol{\mathcal{R}}_n$. (Note that in SN_{SOL} and WN_{SOL}, (BARE) is satisfiable.)

(LL) is not valid in WN_{SOL}*: there may be models with distinct individuals which are in perfect resemblance. The simplest is a model with two individuals and such that, for every adicity n, $\boldsymbol{\mathcal{R}}_n$ only contains $\boldsymbol{\mathcal{I}}^n$. But (LL) is satisfiable.

How could we modify WN_{SOL}* in order that (LL) becomes valid? We would have to impose to a model the following condition: two distinct urelements must not belong to exactly the same members of $\boldsymbol{\mathcal{R}}_1$. (More rigorously, if X_1 and X_2 are two distinct urelements then there must be at least one member Y of $\boldsymbol{\mathcal{R}}_1$ such that Y contains X_1 and not X_2, or Y contains X_2 and not X_1.) The condition concerns only the ontological element of the models itself; thus the claim that the indiscernibles are identical is a genuine ontological claim in WN_{SOL}* (while in SN_{SOL}* it was trivially true).

3.2.4. Is WN_{SOL}* ontologically relevant?

While SN_{SOL}* did not seem to express adequately quantification over properties and relations, I think WN_{SOL}* does so from a weak nominalist point of view. When a weak nominalist is talking about *every property of this apple*, he may mean two different things: 1) he may talk about every (real or not) monadic predicate we can actually apply to this apple; that is the interpretation expressed by WN_{SOL}; 2) since a weak nominalist supposes that the set of individuals is ontologically structured by resemblance and thus certain predicates are *real* predicates, he may also talk about every possible *real* monadic predicates that could apply to this apple; that is the interpretation expressed by WN_{SOL}*. These interpretations are both plausible, but the second one is more appealing since it allows us to express claim concerning the resemblance structure which characterizes the weak nominalist framework; for example it allows us to express that a predicate is real or is not.

Finally, it is worth noting that the ontological commitment of WN_{SOL}* as well as WN_{SOL} is still in strict accordance with weak nominalism: a model is a structure $<\boldsymbol{\mathcal{R}}, \delta>$, where $\boldsymbol{\mathcal{R}}$ is a resemblance structure on individuals and δ a denotation function, and thus every formula of the second-order languages is made true or false only by this resemblance structure and the interpretation of the constants of the language.

3.3. Weak realist second-order logic

3.3.1. A weak realist semantics for second-order languages: WR_{SOL}

The weak realist framework as we defined it earlier is a structure $<\boldsymbol{\mathcal{S}}>$ where $\boldsymbol{\mathcal{S}}$ is a set of couples (intuitively state of affairs), from which we can define a set $\boldsymbol{\mathcal{I}}$ of individuals and a set $\boldsymbol{\mathcal{U}}$ of universals (which is the union of sets $\boldsymbol{\mathcal{U}}_1,\ldots, \boldsymbol{\mathcal{U}}_n$, of universals of different adicity). The first member of every couple in $\boldsymbol{\mathcal{S}}$ is an n-adic universal, and the second member is an n-tuple of individuals. The framework is such that every universal is instantiated and every individual instantiates at least one universal. Individuals and universals are the urelements of the framework. (See 2.3. for all the details.)

As previously and for the same reason, I will not assume that every constant object denotes an individual. I will not assume either that every predicate denotes a universal.

A weak realist model for a second-order language $\mathcal{L}_{\mathcal{V}}$ is a structure <$\mathcal{S}$, δ> satisfying the following conditions: $\mathcal{S}$ is as defined in 2.3, and δ is a denotation function mapping certain constant object of $\mathcal{V}$ to a member of $\mathcal{I}$, and certain *n*-adic constant predicates of $\mathcal{V}$ to a member of $\mathcal{U}_n$ (if $\mathcal{U}_n$ is not empty).

A function *s* is an *assignment* if it satisfies the following conditions: for every variable object *v*, $s(v) \in \mathcal{I}$; for every *n*-adic variable predicate V^n, if $\mathcal{U}_n$ is not empty then $s(V^n) \in \mathcal{U}_n$, and if $\mathcal{U}_n$ is empty then $s(V^n)$ is not defined. In other terms, a value-assignment maps each variable object to an individual (that is standard), and it maps each *n*-adic variable predicate to an *n*-adic universal if there is at least one such universal, otherwise it is not defined.

We define a *denotation function under a given assignment* in the same way as before. We can now define recursively the notion of *truth in a model under an assignment* for any formula of $\mathcal{L}_{\mathcal{V}}$. Let us start with the rule for atomic formulas:

(i) *Atomic formulas.*

$Ms \vDash K^n t_1 \ldots t_n$ iff $\delta_s(t_1), \ldots, \delta_s(t_n)$ and $\delta(K^n)$ are all defined and

$$< \delta(K^n), <\delta_s(t_1), \ldots, \delta_s(t_n)>> \in \mathcal{S}$$

Intuitively, this rule means that $K^n t_1 \ldots t_n$ is true iff the universal denoted by K^n and the individuals denoted by $t_1, \ldots, t_n$ constitute a state of affairs.

The rest of the semantics is the same as in 3.1.1. Truth in a model, satisfiability and validity are also defined as usual. Let us call this theory WR_{SOL}.

3.3.2. Remarkable features of WR_{SOL}

WR_{SOL} is a free logic for constant objects and also for constant predicates: constant objects and predicates may lack denotation. Thus, these two formula schemas are not valid:

(C-IND) $\exists x(x = c)$ where *c* is a constant object

(C-PRED) $\exists F^n(F^n = C^n)$ where C^n is an *n*-adic constant predicate

In WR_{SOL}, an instance of (C-PRED) means: "the constant C^n denotes a universal". (In WN_{SOL}* this same formula means that C^n is a real predicate.)

The formula (ID) is not valid:

(ID) $\exists F^n(F^n = F^n)$

Indeed, in a model where there are no *n*-adic universals, $s(F^n)$ is not defined and thus the formula $F^n = F^n$ is always false since the variables lack denotation. The truth of (ID) for an adicity *n* means that there is at least one *n*-adic universal in the model.

Let us now consider the formulas (ID+COEXT), (UNINST) and (BARE).

While (ID+COEXT) was a valid formula in every nominalist semantics, it is not in WR_{SOL}: two predicates can be coextensive and yet be distinct. Indeed, two predicates can denote two different universals, and therefore be distinct, though those universals are coextensive. However (ID+COEXT) is satisfiable.

(UNINST) is not satisfiable in WR_{SOL}. Indeed, if there is an n-adic universal, then there is a state of affairs in which this universal is instantiated by a n-tuple of individuals. (More formally: in the weak realist framework, the set of universals $\boldsymbol{\mathcal{U}}$ is constructed from the set of state of affairs $\boldsymbol{\mathcal{S}}$, hence every universal appears instantiated in at least one state of affairs.) Thus the negation of (UNINST) is a valid formula of WR_{SOL}.

(BARE) is not satisfiable in WR_{SOL}. Indeed, for a weak realist, every individual appears in at least one state of affairs and therefore it instantiates at least one universal. (More formally: in the weak realist framework, the set of individuals $\boldsymbol{\mathcal{I}}$ is constructed from the set of states of affairs $\boldsymbol{\mathcal{S}}$, hence every individual appears instantiating a universal in at least one state of affairs.) Thus, the negation of (BARE) is valid in WR_{SOL}.

As before, we may note that the logical statuses of (ID+COEXT), (UNINST) and (BARE) in WR_{SOL} express different ontological thesis which are implied by weak realism: two distinct universals can be coextensive, there cannot be uninstantiated predicates and there cannot be bare individuals. Of course, one may say that it does not really tell us anything new about weak realism (and one could make similar remarks concerning the other semantics); but remember that my purpose here is only to formalize in the most rigorous way how second-order formula are made true according to weak realism; this semantics does not show us anything new but it is still interesting in that it gives us a precise and complete picture of the world/language relation according to weak realism. Another interesting aspect also is that we can now deal with ontological questions raised by weak realism using a purely logical device: for example the ontological question "does weak realism implies the impossibility of bare individuals?" becomes the logical question "is the formula (BARE) satisfiable in WN_{SOL}?".

3.3.3. Other ontological claims. Variants of WR_{SOL}

Let us consider the same claims (1)-(4) as before. How should we modify the semantics in order to represent those claims?

(1) Every constant predicate is satisfied.
(2) There are properties.
(3) Every object have at least one property.
(4) Two objects having exactly the same properties are identical.

For (1), we must impose the following condition on the model: every constant predicate denotes a universal. (Indeed, it is a sufficient condition since every universal is instantiated in WR_{SOL}.)

For (2) we only have to specify that $\boldsymbol{\mathcal{U}}_1$ must not be empty.

For (3) the framework must satisfy the following condition: for every X member of $\boldsymbol{\mathcal{I}}$, there is a Y member of $\boldsymbol{\mathcal{U}}_1$ such that the couple $<Y, X>$ is a member of $\boldsymbol{\mathcal{S}}$.

For (4), the framework must be such that two distinct members of $\boldsymbol{\mathcal{I}}$ are not instances of exactly the same members of $\boldsymbol{\mathcal{U}}_1$. (More rigorously: if X_1 and X_2 are two distinct members of $\boldsymbol{\mathcal{I}}$ then there must be at least one member Y of $\boldsymbol{\mathcal{U}}_1$ such that the couple $<Y, X_1>$ is a member of $\boldsymbol{\mathcal{S}}$ and not $<Y, X_2>$, or $<Y, X_2>$ is member of $\boldsymbol{\mathcal{S}}$ and not $<Y, X_1>$.)

Note that (2), (3) and (4) impose purely ontological constraint: we can say that they define variants of the weak realist framework.

3.4. Strong realist second-order logic

3.4.1. A strong realist semantics for second-order languages: SR_{SOL}

I defined a strong realist ontological framework as a structure $<\boldsymbol{\mathcal{U}}, \varepsilon>$ where $\boldsymbol{\mathcal{U}}$ is a set $\{\boldsymbol{\mathcal{U}}_1, \dots \boldsymbol{\mathcal{U}}_n, \dots\}$ whose members are distinct sets of urelements (intuitively they are n-adic universals), and ε is an instantiation function mapping certain members of the $\boldsymbol{\mathcal{U}}_n$*'s* to sets of n-tuples of urelements (all distinct from the members of the $\boldsymbol{\mathcal{U}}_n$'s), and I defined the set of individuals $\boldsymbol{\mathcal{I}}$ as the set of those urelements. (See 2.4 for the details.)

As previously in WR_{SOL}, I will assume that certain constant objects and constant predicates may not denote. I define a strong realist model for a second-order language $\boldsymbol{\mathcal{L}_V}$ as a structure $<\boldsymbol{\mathcal{U}}, \varepsilon, \delta>$ where $\boldsymbol{\mathcal{U}}$ and ε are defined as in 2.4, and δ is a denotation function mapping certain constant objects to a member of $\boldsymbol{\mathcal{I}}$ and certain n-adic constant predicates to a member of $\boldsymbol{\mathcal{U}}_n$.

The notions of *assignment* and *denotation function under an assignment* are defined exactly as in WR_{SOL}. (Intuitively: an assignment maps each variable object to an individual, and it maps each n-adic variable predicate to an n-adic universal if there is at least one n-adic universal, otherwise it is not defined.)

We can now define recursively the notion of *truth in a model under an assignment* for any formula of $\boldsymbol{\mathcal{L}_V}$. First, we give the rule for atomic formulas:

(i) *Atomic formulas.*

$Ms \models K^n t_1 \dots t_n$ iff $\delta_s(t_1), \dots, \delta_s(t_n)$ and $\delta(K^n)$ are all defined and

$$<\delta_s(t_1), \dots, \delta_s(t_n)> \in \varepsilon(\delta(K^n))$$

Intuitively, this rule means that $K^n t_1 \dots t_n$ is true iff the n-tuple of individuals denoted by $t_1, \dots, t_n$ instantiates the universal denoted by K^n.

The other rules are literally the same as in the other semantics. (See 3.1.1 for the details.) Let us call this theory SR_{SOL}.

3.4.2. Remarkable features of SR_{SOL}

SR_{SOL} is a free logic for constant objects and constant predicates (like WR_{SOL}). Thus (C-IND) and (C-PRED) are not valid.

For the same reason as in WR_{SOL}, (ID) and (ID+COEXT) is not valid in SR_{SOL}.

(BARE) is not satisfiable in SR_{SOL} because every individual must appear in the mapping of the instantiation function ε.

(UNINST) is satisfiable (but not valid) in SR_{SOL} because ε does not map *every* universal to a *n*-tuple of individuals. There may be uninstantiated universals.

As before, it is worth noting that the logical statuses of those formulas in SR_{SOL} expresses ontological thesis which are implied by strong realism: two distinct predicates may be coextensive, there may be uninstantiated universals and every individual instantiates at least one universal.

3.4.3. Other ontological claims. Variants of SR_{SOL}

Let us consider the claims (1)-(4) as in 3.1.3 and 3.3.3. How should we modify the semantics in order to represent those claims?

For (1) we must impose the following condition on models: every constant predicate denotes a universal, and for every member X of a set $\mathcal{U}_n$, if there is a Y member of $\mathcal{V}_n$ such that $\delta(Y) = X$, then $\varepsilon(X)$ is a non-empty set. (The condition is more complex than the one in WR_{SOL} because in WR_{SOL} there are no uninstantiated universals, therefore if a constant predicate denotes a universal then this constant predicate is instantiated. In SR_{SOL}, even if a constant predicate denotes an universal, it is not guaranteed that this universal is instantiated.)

For (2) We only have to specify that $\mathcal{U}_1$ must not be empty. It is the same as in WR_{SOL}, but note that it does not assure that there are *instantiated* properties: there may be properties but only uninstantiated ones.

For (3) the framework must satisfy the following condition: for every X member of $\mathcal{I}$, there is a Y member of $\mathcal{U}_1$ such that X is a member of $\varepsilon(Y)$.

For (4) the framework must be such that two distinct members of $\mathcal{I}$ are not instances of exactly the same members of $\mathcal{U}_1$. (More rigorously: if X_1 and X_2 are two distinct members of $\mathcal{I}$ then there is a Y member of $\mathcal{U}_1$ such that X_1 is a member of $\varepsilon(Y)$ and not X_2, or X_2 is a member of $\varepsilon(Y)$ and not X_1.)

As for WR_{SOL}, we may observe that (2), (3) and (4) impose purely ontological constraint: they define variants of the strong realist framework.

3.5. Tropist second-order logic

3.5.1. A tropist semantics for second-order languages: T_{SOL}

Let us briefly recall what is the tropist framework as it is defined in 2.5. It is a structure $<\mathcal{C}, \mathcal{R}>$ satisfying the following condition. First, $\mathcal{C}$ is a set of non-empty disjoint sets of urelements (intuitively it is a compresence structure on tropes). Then we define the set of tropes $\boldsymbol{\mathcal{T}}$ as the union of the members of $\boldsymbol{\mathcal{C}}$. Finally, we have to define $\boldsymbol{\mathcal{R}}$. Intuitively, it is a set whose members are sets of resembling tropes, either monadic or polyadic. Remember that we have defined an n-adic tropes for $n \geq 2$ as n-tuples of monadic tropes. The set $\boldsymbol{\mathcal{R}}$ must be such that: i) if a trope appears in an n-adic trope then it appears only in this n-adic trope; ii) each trope resembles at least itself. (See 2.5 for formal details.)

Now, I will define a semantics for second-order languages in this tropist framework. As previously, I will not suppose that constant objects and constant predicates always have a denotation.

Let us call this last theory T_{SOL}. A model of T_{SOL} for a second-order language $\boldsymbol{\mathcal{L}_{\mathcal{V}}}$ is a structure $< \boldsymbol{\mathcal{C}}, \boldsymbol{\mathcal{R}}, \delta>$ where $\boldsymbol{\mathcal{C}}$ and $\boldsymbol{\mathcal{R}}$ (and therefore also the set $\boldsymbol{\mathcal{T}}$ of tropes) are defined as in 2.5, and δ is a denotation function mapping certain constant objects of $\boldsymbol{\mathcal{V}}$ to a member of $\boldsymbol{\mathcal{C}}$, and certain n-adic constant predicates of $\boldsymbol{\mathcal{V}}$ to a member of $\boldsymbol{\mathcal{R}}$ whose members are sets of n-tuples of tropes. Intuitively: a constant object may denote a set of compresent tropes, and an n-adic constant predicate may denote a set of resembling n-adic tropes.

An assignation function s is defined as follows: a function s is an assignation if it maps every variable object to a member of $\boldsymbol{\mathcal{C}}$ and every n-adic variable predicates to a member of $\boldsymbol{\mathcal{R}}$ whose members are sets of n-tuples of tropes if there is such a member of $\boldsymbol{\mathcal{R}}$, otherwise it is not defined. (Thus, $s(F^n)$ is not defined if there are no n-adic tropes.)

We define as usual a *denotation function under a given assignment*. And we can finally define recursively the notion of *truth in a model under an assignment* for any formula of $\boldsymbol{\mathcal{L}_{\mathcal{V}}}$.

(i) *Atomic formulas.*
$Ms \vDash K^n t_1 ... t_n$ iff $\delta_s(t_1)$, ..., $\delta_s(t_n)$ and $\delta(K^n)$ are all defined and there are X_1, ..., X_n such that $X_1 \in \delta_s(t_1)$, ..., $X_n \in \delta_s(t_n)$ and $<X_1, ..., X_n> \in \delta(K^n)$.

Intuitively, this rule means that $K^n t_1 ... t_n$ is true iff there are n tropes X_1, ..., X_n such that each one belongs respectively to the set of compresent tropes denoted respectively by t_1, ..., t_n, and the n-tuples $<X_1, ..., X_n>$ is an n-adic trope belonging to the set of resembling n-adic tropes denoted by K^n.

The other rules are the same as before. (In fact, concerning the semantic rules, all the semantics we have seen in this paper differ only with respect to the rule for atomic formulas.)

Truth in a model, satisfiability and validity are also defined as usual.

3.5.2. Remarkable features of $\boldsymbol{T_{SOL}}$

T_{SOL} is a free logic for constant objects and constant predicates (like WR_{SOL} and SR_{SOL}). Thus (C-IND) and (C-PRED) are not valid.

The assignation of an n-adic variable predicate F^n is not defined when there are no n-adic tropes, and a tropist framework may not contain tropes of every adicity. Thus, (ID) is not valid.

(ID+COEXT) also is not valid (though it is satisfiable). Predicates may denote distinct sets of resembling tropes which belongs to the same sets of compresent tropes. Consider for example a world where the only sets of compresent tropes (i.e. the only individuals) are two red round things: one is constituted of a trope of Red_1 and a trope of $Round_1$, the second is constituted of a trope of Red_2 and a trope of $Round_2$. And assume that there are only two sets of resembling tropes in this world (i.e. two universals): one consists of the tropes of Red_1 and Red_2, the other of the tropes of $Round_1$ and $Round_2$. These two universals are coextensive and yet distinct.

(UNINST) is not satisfiable. Indeed, the $\boldsymbol{\mathcal{R}}_n$'s cannot contain the empty set, therefore a denoting predicate will always denote a set containing at least one trope; this trope necessary belongs to a set of compresent tropes; and this set constitutes an individual to which this predicate applies.

One may wonder why a $\boldsymbol{\mathcal{R}}_n$ cannot contain the empty set while in the weak nominalist framework we allow the empty set to appear among the resemblance sets. Well, the difference is that in a tropist framework, the sets of resembling tropes are supposed to stand for universals, and it would seem to me very dubious that the empty set may stand for a universal. (In the weak realist framework, the sets of resembling individuals stand for the *extensions* of predicates, that is very different; it seems to me rather natural that the empty set may be the *extension* of a predicate.)

(BARE) is not satisfiable. Indeed, every individual is constituted of at least one trope (the compresence structure represented by $\boldsymbol{\mathcal{C}}$ is a set of *non-empty* disjoint sets of tropes), and every trope resemble at least itself; thus every individual instantiates at least one universal.

(LL) is not valid but it is satisfiable. (If we want (LL) to be valid, we should impose the following condition on models: two distinct members of $\boldsymbol{\mathcal{C}}$ must not overlap exactly the same members of $\boldsymbol{\mathcal{R}}$.)

3.6 Comparison and conclusion

We can sum up in a table the differences between the seven semantics we have constructed.

'*V*' means that the formula is valid (*generally* valid).

'*OV*' means that the formula is *occasionally* valid (i.e. valid for certain languages but not all).

'*S*' means that the formula is not valid but is satisfiable (true in some models but not all).

'U' means that the formula is not satisfiable (therefore its negation is valid).

	SN_{SOL}/ WN_{SOL}	SN_{SOL}*	WN_{SOL}*	WR_{SOL}	SR_{SOL}	T_{SOL}
(ID)	*OV*	*V*	*V*	*S*	*S*	*S*
(C-PRED)	*V*	*V*	*S*	*S*	*S*	*S*
(ID+COEXT)	*V*	*V*	*V*	*S*	*S*	*S*
(UNINST)	*S*	*V*	*S*	*U*	*S*	*U*
(BARE)	*S*	*U*	*U*	*U*	*U*	*U*
(LL)	*S*	*V*	*S*	*S*	*S*	*S*

We have now a clearer view of how the disagreements between strong nominalism, weak nominalism, weak realism, strong realism and tropism, extend from ontology to logic. Endorsing an ontological view or another has (or should have) an impact on the logic we use.

However, a careful reader may have noticed that the table shows no difference between T_{SOL} and WR_{SOL}. Maybe those two semantics are logically equivalent; I have not studied this question thoroughly. Anyway, they differ greatly from an ontological point of view: the two frameworks are not the same at all, the same formula does not mean the same thing in a semantics and in the other, and the way a formula is made true in one semantics is very different from the way the same formula is made true in the other semantics.

Let us take for example the formula (BARE) and see what it means in those different semantics. In SN_{SOL} and WN_{SOL} it means that there is an individual that does not satisfy any constant predicate of the language. In SN_{SOL}* it means that there is an individual that does not belong to any set of individual. In WN_{SOL}* it means that there is an individual that does not belong to any set of resemblance. In WR_{SOL} and SR_{SOL} it means that there is an individual that does not instantiate any universal. And in T_{SOL} it means that there is a set of compresent tropes such that none of those tropes belong to a set of resembling tropes. Those are five very different readings.

A first interesting result of this study of second-order logics is that we have seen that we can perfectly interpret second-order quantification as quantification over properties and relations without being ontologically committed to universals: see SN_{SOL} and WN_{SOL}. More generally, I hope it is now clear why, when we interpret a certain language in a certain ontological framework, the features of the language cannot change anything to the ontological commitments of the semantics: *the ontological commitments are only determined by the ontological framework from which we construct the semantics.*

There is another important aspect of this study: though we have not learned anything new about those five different ontological views, we have formalized in the most rigorous way the world/language relation (and notably the truthmaking relation) according to these different views; and various metaphysical questions can now be treated by pure logic in those frameworks. I think it is good enough to show that the method I have been using here is an interesting way to do metaphysics.

4. Other perspectives: interpreting quantified modal logic

The method I have exposed and applied for second-order logic could be used for the construction of semantics for any other kind of language; for example we could consider quantified modal languages. As a conclusion to this paper, I will make few remarks about how we could construct such semantics.

There are various ways of interpreting quantified modal languages. One of the best known is Lewis' modal realism according to which there is a plurality of worlds, all isolated from each other; a world is made of nothing but individuals and each individual belongs to only one world. Though Lewis endorses nominalism, a lewisian framework would be different from the strong or weak nominalist framework we have defined. A lewisian framework could be for example a structure $<\boldsymbol{\mathcal{W}}>$ where $\boldsymbol{\mathcal{W}}$ is a set of non-empty disjoint sets of urelements, and we define $\boldsymbol{\mathcal{I}}$ as the set of those urelements; intuitively, $\boldsymbol{\mathcal{W}}$ stands for the set of all possible worlds: thus each possible world is constituted of a set of individuals, and each individual only belongs to one and only one world. Of course, this framework is not yet adequate to represent Lewis' theory, but my point here is that modal realism is a an ontological view that requires the construction of another kind of ontological framework.

Could we construct a semantics for quantified modal logic from one of the five ontological frameworks defined in section 2? We must note that the individuals of the strong and weak nominalist frameworks, the individuals and universals of the weak and strong realist frameworks, and the tropes of the tropist framework, are supposed to be actual entities (actual individuals, universals and tropes); in none of these frameworks it seems that we can find *possibilia* or *possible worlds*. How are we going to interpret modal formulas?

The solution is to construct ersatz of possible worlds from the actual entities of the framework. It is obviously an actualist solution. In fact we can know *a priori* that if one achieves the construction of an appropriate semantics for quantified modal languages from one of those five frameworks, it will be the expression of an actualist theory of possibility: those frameworks only contain actual entities, therefore only actual entities will make true the modal formulas interpreted in any semantics constructed from those frameworks, and thus we will be ontologically committed only to actual entities.

Let us finally give a sketch of such a semantics. One could construct a weak realist semantics for quantified modal languages inspired by the theory set forth by Armstrong [13]. It is a combinatorial theory of possibility. Basically, we start from a world of (actual) states of affairs; we assume that any combination of an n-adic universal with n individuals stands for a possible state of affairs; and any set of possible states of affairs stands for a possible world. These ideas can be thoroughly expressed within the weak realist framework $<\boldsymbol{\mathcal{S}}>$. (See 2.3. Each member of $\boldsymbol{\mathcal{S}}$ represents a state of affairs as a couple whose first member is an n-adic universal and the other is an n-tuple of individuals; the set $\boldsymbol{\mathcal{U}}$ is defined as the set $\{\boldsymbol{\mathcal{U}}_1, \ldots, \boldsymbol{\mathcal{U}}_n, \ldots\}$ whose members are sets of n-adic universals, and $\boldsymbol{\mathcal{I}}$ is defined as the set of individuals.) We can define the set $\boldsymbol{\mathcal{S}}^*$ of possible states of affairs as the set of couples $<X, Y>$ such as $X \in \boldsymbol{\mathcal{U}}_n$ and Y is an n-tuple $<Y_1, \ldots, Y_n>$ such as $Y_1 \in \boldsymbol{\mathcal{I}}, \ldots, Y_n \in \boldsymbol{\mathcal{I}}$. And we can define the set of worlds $\boldsymbol{\mathcal{W}}$ as the power set of $\boldsymbol{\mathcal{S}}^*$. (This is an imperfect sketch. In fact, we should add several constraints on the construction of possible worlds in order to fit exactly with Armstrong's view, but it gives a first idea of how we can do it.) It is very important to note that $\boldsymbol{\mathcal{S}}^*$ and $\boldsymbol{\mathcal{W}}$ do not add anything to the ontological framework: they both have been constructed from $\boldsymbol{\mathcal{S}}$ and nothing more. Then we can use this set of worlds for the interpretation of quantified modal languages in the usual way. Since the construction of those worlds is entirely determined by the actual states of affairs, what makes true the modal formulas would be *in fine* nothing more but actual states of affairs: the ontological commitments of the semantics are only determined by the ontological framework from which it is constructed.

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